## 1. Proof by cases

Recall that a deductively valid argument has the following property: if its premises are true, then its conclusion must be true. A classic example of a schema for producing valid arguments is known as *proof by cases* (also known as *constructive dilemma*). A proof by cases has this form:

1. Either P or Q. (Premise)
2. If P, then R. (Premise)
3. If Q, then R.(Premise)
4. Therefore R. (From 1–3)

We can imagine any number of arguments fitting this pattern. For instance,

1. Either my mom is home or my dad is home.
2. If my mom is home, then at least one of my parents is home.
3. If my dad is home, then at least one of my parents is home.
4. So at least one of my parents is home.

Here is another example:

1. Either Barack will win or Hillary will win.
2. If Barack wins, then a conservative will win.
3. If Hillary wins, then a conservative will win.
4. Therefore a conservative will win.

And here is another example:

1. Either India will attack or Pakistan will attack.
2. If India attacks, then catastrophe will ensue.
3. If Pakistan attacks, then catastrophe will ensue.
4. Therefore catastrophe will ensue.

## 2. Henry's speech before the Battle of Agincourt

In 1415 in northern France, King Henry V of England led his badly outnumbered army against the French army, resulting in the Battle of Agincourt. The battle has since become legendary and it forms a centerpiece in Shakespeare's historical play *Henry V*.

In Act 4, Scene 3, Shakespeare presents the following dialogue, where Henry is trying to rally the troops in the face of long odds.

*Westmoreland:*
> O that we now had here
> But one ten thousand of those men in England
> That do no work today!

*King Henry V:*
> What's he that wishes so?
> My cousin Westmoreland? No, my faire cousin:
> If we are mark'd to die, we are enow
> To do our country loss; and if to live,
> The fewer men, the greater share of honour.
> God's will! I pray thee, wish not one man more.

This persuades Westmoreland, for moments later they have this exchange:

*King Henry V:*
> Thou dost not wish more help from England, coz?

*Westmoreland:*
> God's will! my liege, would you and I alone,
> Without more help, could fight this royal battle!

## 3. A successful argument?

Given the poetic license on display in this dialog, there is bound to be more than one interpretation of Henry's reasoning in response to Westmoreland's initial wish for reinforcements. But here's one way to understand it.

1. Either we will win or we will lose. (Premise)
2. If we win, having fewer is better. (Premise)
3. If we lose, having fewer is better. (Premise)
4. Therefore having fewer is better. (From 1–3)

Are the premises true? It certainly seems that they are. By 'lose' we just mean 'not win', so line 1 is true. Line 2 seems true, because winning with fewer is more efficient and more glorious. (The English did win, by the way, which is why the battle is legendary.) Line 3 seems true, because losing with fewer brings with it no shame, and decreases overall loss and suffering.

Is the argument valid? Notice what form this argument has: proof by cases. In light of that fact, we ought to consider it valid. But there's something odd about this. It's odd because the conclusion seems *obviously false*, whereas the premises seem *true*. But if the argument is valid, then it's impossible for the premises to be true while the conclusion is false!

Something has to give. But what? We have three options:

A) Deny that the argument is valid, despite appearances.
B) Deny one of the premises, despite appearances.
C) Accept the argument's counterintuitive conclusion.

We could effectively implement option A by locating an ambiguity in the argument (resulting in a fallacy of equivocation). We could effectively implement option B by stating a good reason to reject one or more premises. We could effectively implement option C by stating a good reason to accept the conclusion. What's the best way forward?